

Mathematics Without Frontiers



Competition 10 February 2015

- ✓ Hand in only one answer sheet for each question.
- ✓ Even partial solutions and attempts can gain marks.
- ✓ Neat and careful work is important.

Question 1
7 marks

Antigone's polygons

Solution à rédiger en allemand, anglais, espagnol ou italien en un minimum de 30 mots.

Nachdem Antigone ein Dreieck, ein Viereck und ein Fünfeck gezeichnet hat, stellt sie fest, dass ein Dreieck keine, ein Viereck zwei und ein Fünfeck fünf Diagonalen besitzt.

Sie fragt sich, wie viele Diagonalen wohl ein Sechseck, ein Siebeneck und ein Achteck haben.

Sie glaubt, eine Formel gefunden zu haben, die die

Anzahl der Diagonalen in einem n -Eck angibt: $\frac{n(n-3)}{2}$.

Wie viele Diagonalen besitzt ein Sechseck, wie viele ein Siebeneck und wie viele ein Achteck?

Beweist die Formel, die Antigone gefunden hat.

Kann ein Vieleck 100 Diagonalen besitzen? Begründet eure Antwort.

Après avoir fait les figures, Antigone remarque qu'un triangle n'a pas de diagonale, qu'un quadrilatère en a deux et qu'un pentagone en a cinq.

Elle cherche combien de diagonales ont les polygones de 6, 7 et 8 sommets. Elle pense avoir trouvé une formule donnant le nombre de diagonales d'un polygone à n sommets :

$$\frac{n(n-3)}{2}$$

Combien de diagonales ont les polygones à 6, 7 et 8 sommets ?

Démontrer la formule trouvée par Antigone.

Est-il possible qu'un polygone ait 100 diagonales ? Expliquer.

Después de dibujar las figuras, Antigono se da cuenta que los triángulos no tienen diagonales, que los cuadriláteros tienen dos y que los pentágonos tienen cinco.

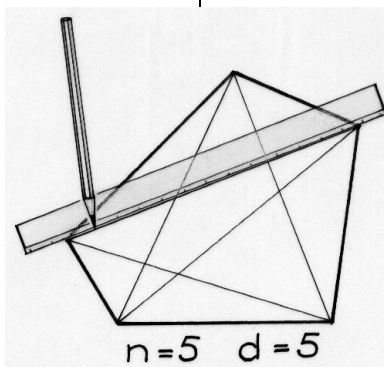
Busca cuántas diagonales tienen los polígonos de 6, 7 y 8 vértices. Antigono piensa que ha encontrado la fórmula que expresa el número de diagonales de un polígono de

$$n \text{ vértices: } \frac{n(n-3)}{2}$$

¿Cuántas diagonales tienen los polígonos de 6, 7 y 8 lados?

Demuestra la fórmula que ha encontrado Antigono.

¿Puede tener un polígono 100 diagonales? Justifica la respuesta.



Dopo aver tracciato le figure, Antigone nota che un triangolo non ha alcuna diagonale, un quadrilatero ne ha due e un pentagono ne ha cinque.

Antigone ricerca quante diagonali possano avere i poligoni rispettivamente con 6, 7 e 8 vertici.

Pensa di avere individuato la formula che indica quante diagonali ha un poligono di n vertici: $\frac{n(n-3)}{2}$.

Quante diagonali hanno i poligoni di 6, 7 e 8 vertici?

Dimostrare la formula individuata da Antigone.

E' possibile che un poligono abbia 100 diagonali? Spiegare.

Question 2
5 marks

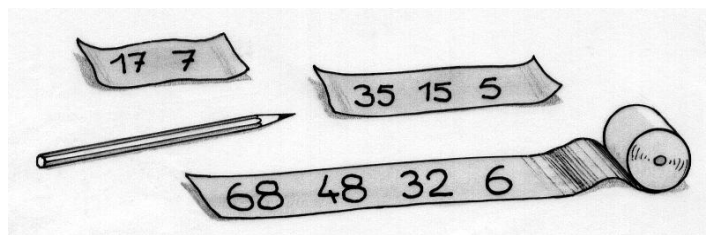
Planned obsolescence

Anne Marie is having some fun playing around with number sequences.

She chooses a whole number as the first number in the sequence. She works out the next one in the sequence by multiplying the digits.

She does the same thing again with the result and continues until she has a number with only one digit.

For example, if she starts with 68, she gets the sequence 68, 48, 32, 6.



What is the whole number less than 100 that gives the longest sequence when you follow this process ?

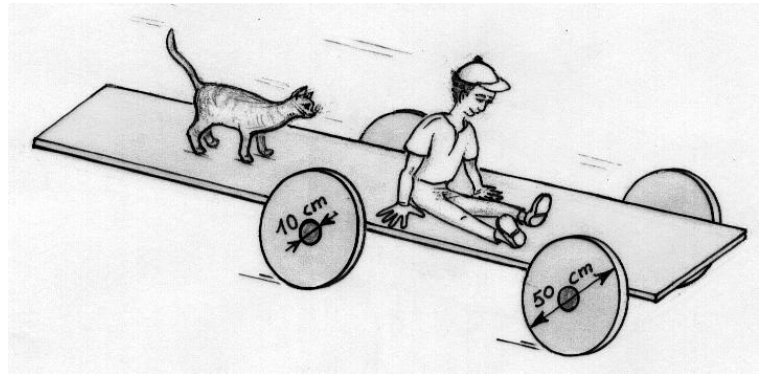
Question 3
7 marks

Quite a turn

A long rigid plank rests on two axles. The axles are parallel as shown. The wheels roll without slipping and the plank moves forward without slipping on the axles.

The diameter of the axles is 10 cm and the wheels attached to the axles have a diameter of 50 cm.

What distance does the plank move when the wheels turn round once? Explain your answer.



Question 4
5 marks

Four by four by ...

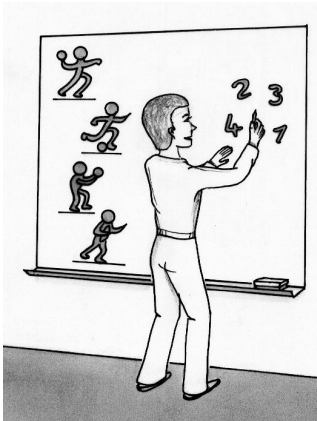
The city of Coubertin is organising a sports tournament.

There are four sports on the programme: volley-ball, football, handball and rugby.

The rules state:

- Each team has to play exactly four matches, one in each of the four sports;
- No team can meet the same team twice.

Show how the tournament could be organised if eight teams are taking part.



Question 5
7 marks

Try-a-circle

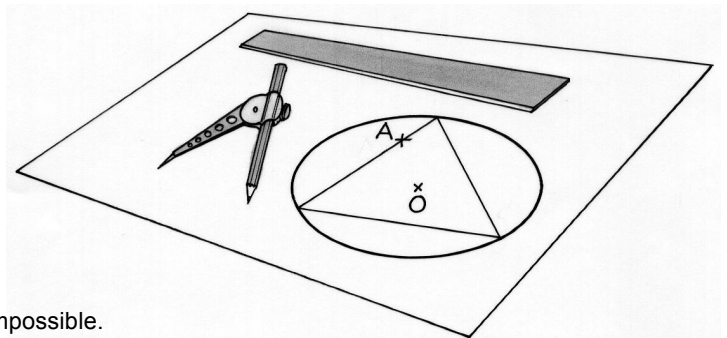
Draw a circle radius 6 cm. Mark in it a point A, 5 cm from the centre.

Inside the circle construct an equilateral triangle whose vertices touch the circle. One of the triangle's sides must pass through the point A.

Write down the stages in your construction.

If the point A is too close to the centre this construction is impossible.

Find and show in colour the locus of points for which the construction is possible.

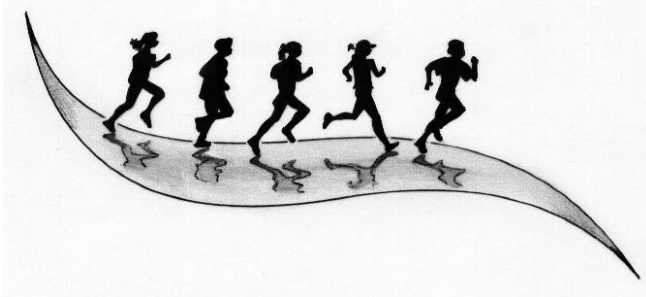


Question 6
5 marks

Five of clubs

Ahmed, Bénédicte, Cyrielle, Damien and Elise take part in a team race for their running club. They start off together and reach the finish line in alphabetical order at 5 minute intervals. Ahmed runs 5 times faster than Elise.

Find the total time taken by the five club members.

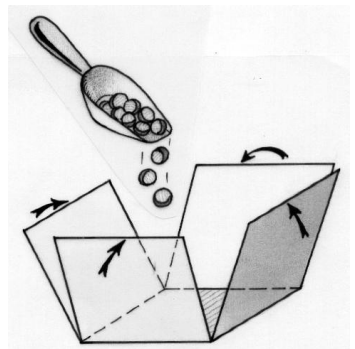


Question 7
7 marks

Boxing clever

Starting with a circle of cardboard 10 cm in radius I construct the net of an open box. The net is made up of five identical squares. With the net I can make a cubical box without a lid.

Calculate the volume of the biggest box I can make.



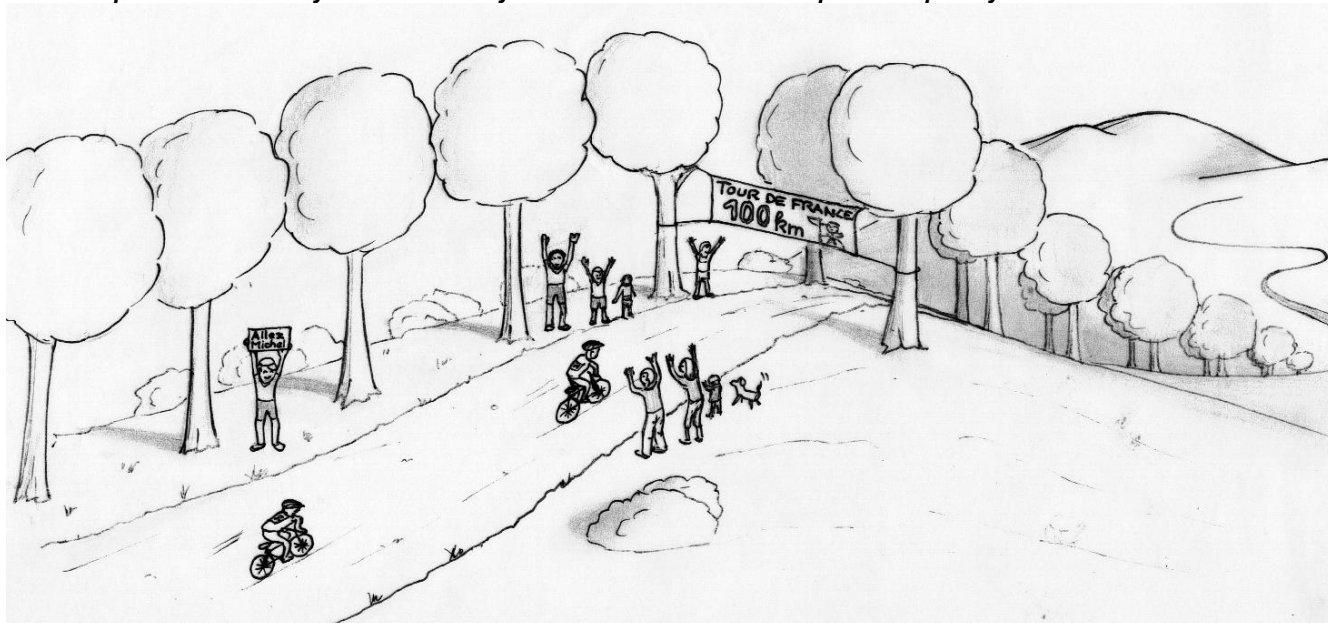
Question 8
5 marks

Breakaway

During the ascent of a steep hill two cyclists break away from the pack ; they are both going at a constant speed of 18 km/h. They are 200 m apart.

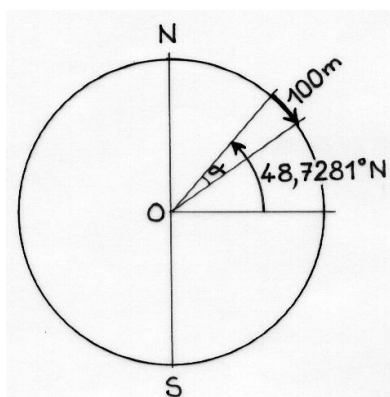
They reach the summit and start the descent. After leaving the summit they both take the same time and cover the same distance before reaching their same constant speed of 70 km/h.

How far apart are the two cyclists when they are both at that constant speed? Explain your answer.



Question 9
7 marks

Too much latitude



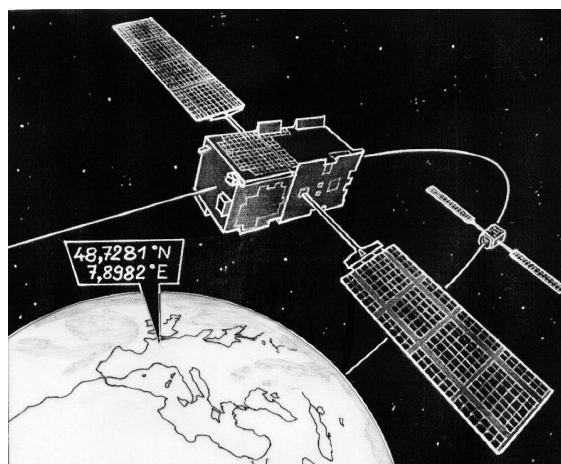
To calculate a position on the surface of the earth a GPS works out the angular position, that is latitude and longitude, using several satellites.

I am at the point with coordinates 48.7281° latitude North and 7.8982° longitude East.

I move 100 metres South keeping the same longitude.

The Earth can be considered to be a sphere of radius 6 367 km.

The diagram shows how latitude is measured.

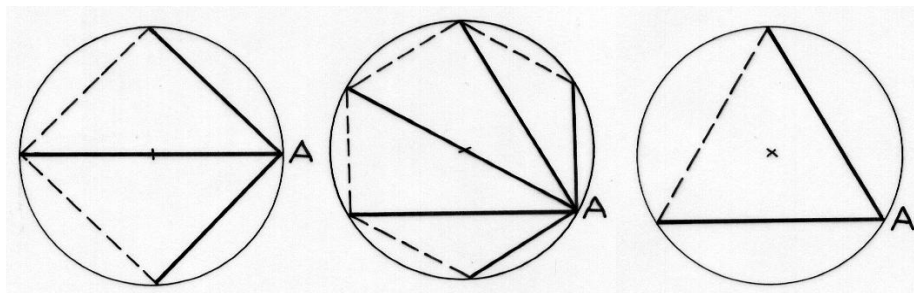


What is the latitude of my new position given by the GPS? Justify your answer.

Question 10
10 marks

In theory

In the diagram regular polygons have been inscribed in circles of radius 1. The thicker lines are chords of the circle and link the vertex A to the other vertices.



For these three regular polygons calculate the exact value of the product of the lengths of the chords drawn from A.

Using the three examples make a conjecture about similar products. According to your conjecture what will be the value of the product for a regular chiliagon (a polygon with 1000 sides)?

SENIOR CLASSES ONLY

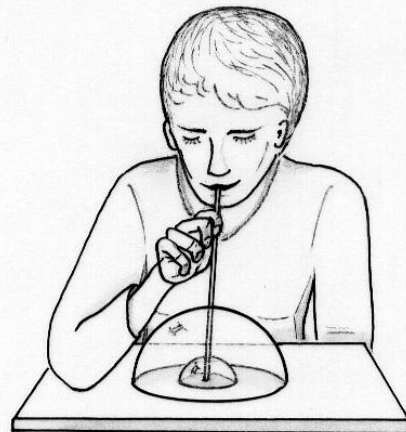
Question 11 5 marks

In soapy bubble

By blowing gently through a straw soaked in soapy water Estelle can make a soap bubble in the shape of a hemi-sphere. The bubble sits on the horizontal surface and has a diameter of 12 cm.

She blows a second bubble inside the first. The first bubble then gets bigger. Its final volume is the sum of its initial volume plus the volume of the bubble inside.

What will be the diameter of the inside bubble when the diameter of the bigger one is 14 cm. Justify your answer.



Question 12 7 marks

Landing area

At their after-school maths club a group has constructed a truncated tetrahedron out of cardboard. Truncated means "with bits cut off" and here four mini-tetrahedra have been cut away as shown.

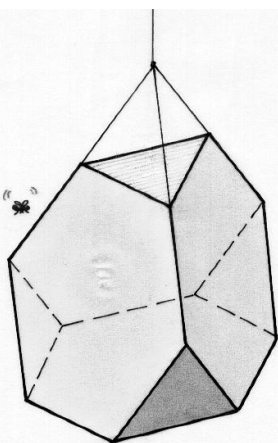
Their model has:

- 4 faces that are regular hexagons
- 4 faces that are equilateral triangles

It is hung up from the ceiling in the classroom.

A fly lands at random on a point on the surface of the model. (The fly does not land on an edge.)

Calculate the probability that the fly lands on one of the hexagonal faces.



Question 13 10 marks

Wipe out ?

Éric asks a car manufacturer to test his prototype windscreen wiper. He has designed it for flat windscreens.

The wiper BC is attached on to the side BB' of the parallelogram. The side AA' is fixed to the car and does not move.

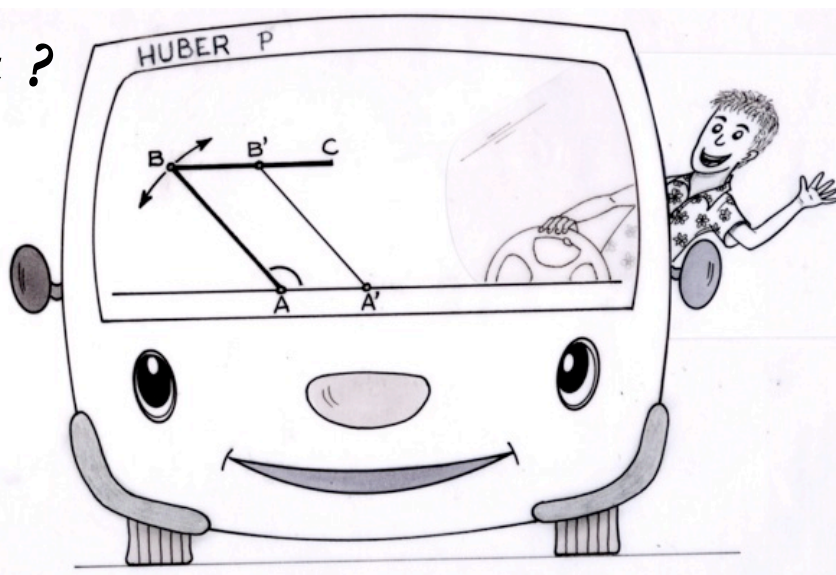
AB = BC = 70 cm. B' is the midpoint of BC.

The angle A'AB of the parallelogram as the wiper moves can vary from 0° to 180° .

Draw and highlight in colour the area of the windscreen cleaned by the wiper blade, BC. Use a scale of 1/10.

Calculate this area.

Will the car manufacturer use Eric's design? Explain your answer.



Question 13 Pro 10 marks

All will be well

Manon and her brothers want to share out a square area of land in a fair way. The square has side 120m. Each one has to have the same area of land. Manon makes her choice first and picks the shaded area on the plan. On Manon's area of land there is a well. Each of her brothers wants to make sure that his piece of land has access to Manon's so that they have access to water. The areas of land can meet along a side or just at a vertex.

Find the number of brothers that Manon has and suggest a way of sharing the ground. Justify your answer.

You can use a geometry software package.

