North Lanarkshire Council

HERIOT WATT UNIVERSITY

ACADÉMIE DE STRASBOURG

2013

Competition
March 2013

Even partial solutions and attempts can gain marks.
Neat and careful work is important.
Hand in only one team answer sheet for each question.

Question 1 Clown aveugle

7 MARKS

Give your answer in French, German, Italian or Spanish using at least 30 words.

Trois clowns, Anatole, Michel et Thomas, ont déposé trois chapeaux rouges et deux chapeaux verts dans leur loge.
Avant d'entrer en scène, ils doivent récupérer chacun un chapeau.
Les clowns ne trouvent pas l'interrupteur et la loge est plongée dans le noir. Chacun prend un chapeau au hasard et le pose sur sa tête. Ils sortent de la loge et entrent en scène.
On demande à chaque clown s'il est capable de deviner la couleur de son chapeau.
Anatole regarde les deux autres et dit « Non ».
Puis Michel regarde les deux autres et dit « Non ».
Enfin Thomas, qui est aveugle, répond « Oui ».

Expliquer comment ce clown aveugle a pu déterminer la couleur de son chapeau.

Drei Clowns, Anatole, Michel und Thomas, haben drei rote Hüte und zwei grüne Hüte in ihrer Garderobe
Vor ihrem Auftritt muss jeder der drei Clowns einen Hut holen.
Die Clowns finden den Lichtschalter nicht und in der Garderobe ist es dunkel. Jeder nimmt zufällig einen Hut und setzt ihn auf. Sie gehen aus der Garderobe hinaus und treten auf.

Question 1 Foreign language question

7 MARKS

Jeder Clown wird gefragt, ob er in der Lage ist, die Farbe seines Hutes zu erraten.
Anatole schaut die beiden anderen an und sagt: „Nein".
Dann schaut Michel die beiden anderenan und sagt: „Nein".
Zuletzt antwortet Thomas, der blind ist: „Ja".

Erklärt, wie der blinde Clown die Farbe seines Hutes bestimmen konnte.

Tre clown, Anatole, Michele e Tommaso hanno depositato in camerino tre cappelli rossi e due verdi.
Prima di entrare in scena ognuno di loro deve recuperare un cappello.
I clown non trovano l'interruttore e il camerino è completamente al buio. Tutti prendono un cappello a caso, se lo mettono, poi, escono dal camerino ed entrano sul palcoscenico.
Alla domanda se sono in grado d'indovinare il colore del proprio cappello.
Anatole guarda gli altri due e dichiara : « No ».
Michele, a sua volta, guarda gli altri due e dichiara : « No ».
Tommaso, infine, che è cieco risponde : « Sì ».

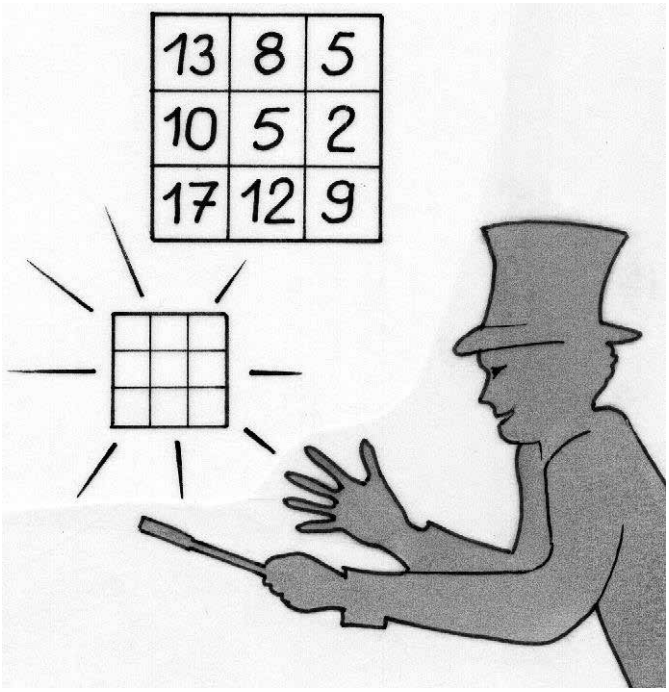
Spiegate come il clown cieco abbia potuto determinare il colore del suo cappello.

Tres payasos, Anatole, Michel y Thomas, han dejado tres sombreros rojos y dos sombreros verdes en el camerino.
Antes de salir a escena, tienen que coger un sombrero cada uno.
Los payasos no encuentran el interruptor y el camerino está a oscuras. Cada uno coge un sombrero al azar y se lo pone en la cabeza. Salen del camerino y entran en escena.
Preguntamos a cada payaso si es capaz de adivinar el color de su sombrero.
Anatole mira los otros dos y dice "No".
Luego Michel mira los otros dos y dice "No".
Por fin Thomas, que es ciego, dice "Si".

Explica cómo el payaso ciego ha podido adivinar el color de su sombrero.

Question 2 Mathemagic

5 MARKS



This is the magic square!
Pick any three numbers from the square. But! - no two numbers can be in the same row or column. Write down the sum of the three numbers.

Do this again using different numbers from the square but keeping to the same restriction.

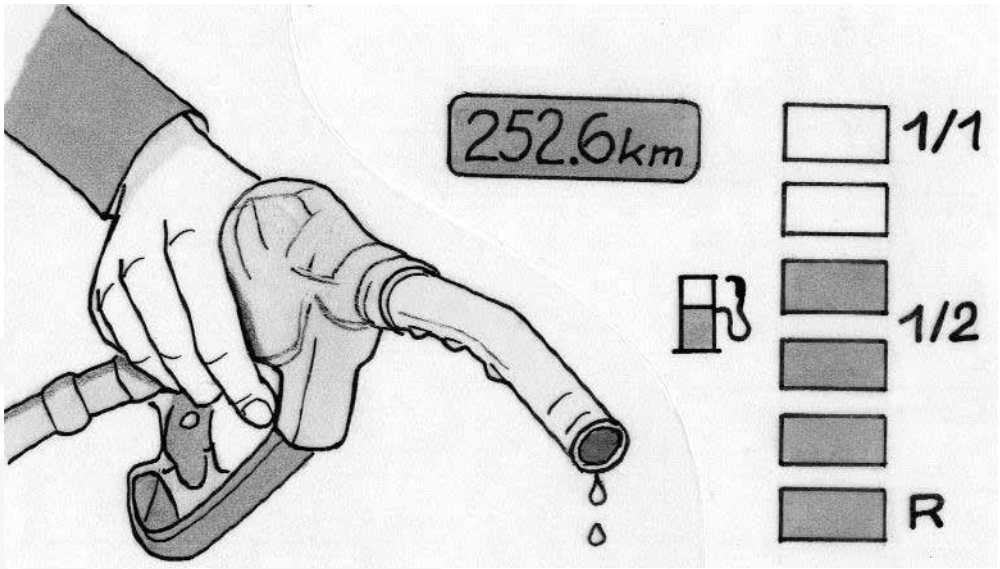
Why can you call this square magic?
Make another magic square of nine squares. Make the sum of three numbers equal to 40. The square must use 9 different numbers.

Question 3 Reserved matter

7 MARKS

Every time I put petrol in my car I fill the tank completely and put the trip counter back to zero.
On the dashboard the volume of the petrol tank is shown as six rectangles. Each rectangle represents a sixth of the volume of the tank.
When a sixth of the tank has been used the black rectangle turns white.
When the fifth rectangle turns white there is an audible signal and the last rectangle starts to blink. At that point the car is running on its "reserve" R.
Since the last petrol fill-up the car has gone 252.6 km and there are still 4 black rectangles.

Calculate the minimum and maximum distances that I can hope to travel in the same traffic conditions before I am using the reserve.

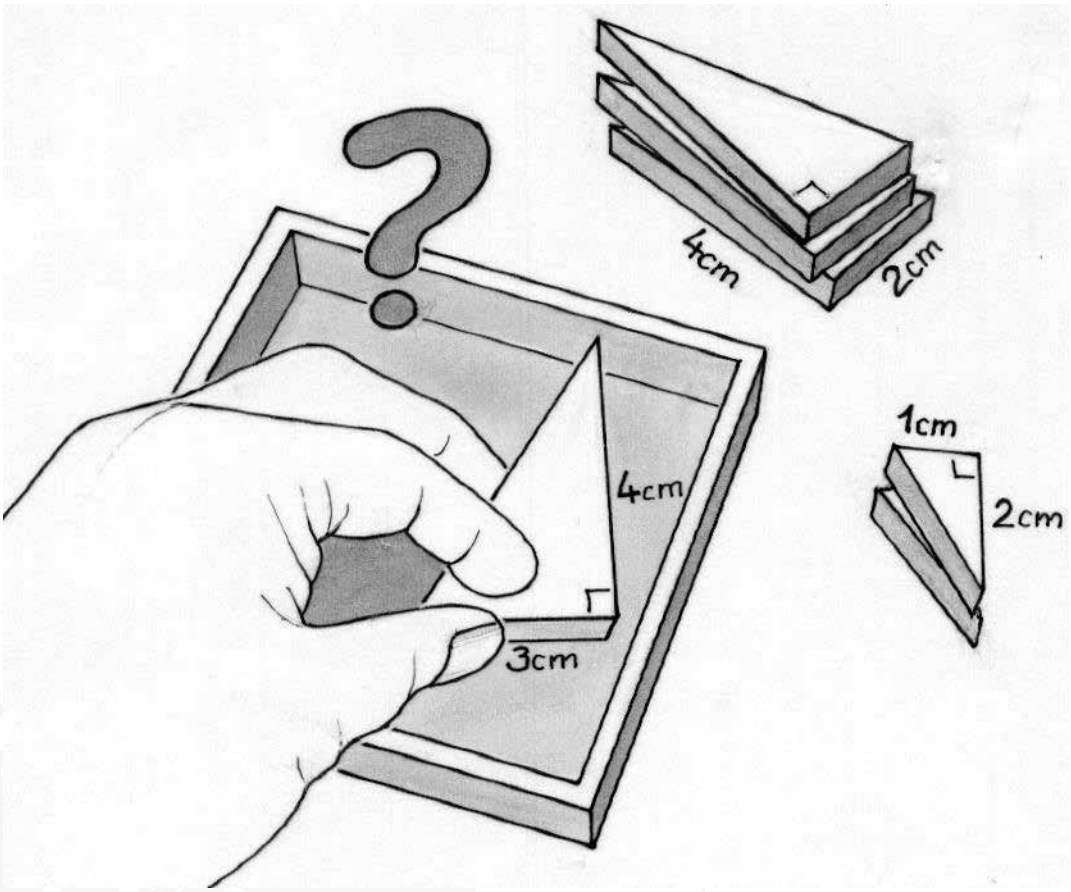


Question 4 A square go

5 MARKS

Make a square with the 6 right-angled triangles shown:

- **2 triangles where the sides forming the right angles are 1 and 2 cm;**
- **3 triangles where the sides forming the right angles are 2 and 4 cm;**
- **1 triangle where the sides forming the right angles are 3 and 4 cm.**



Question 5 Equality, Fraternity .....

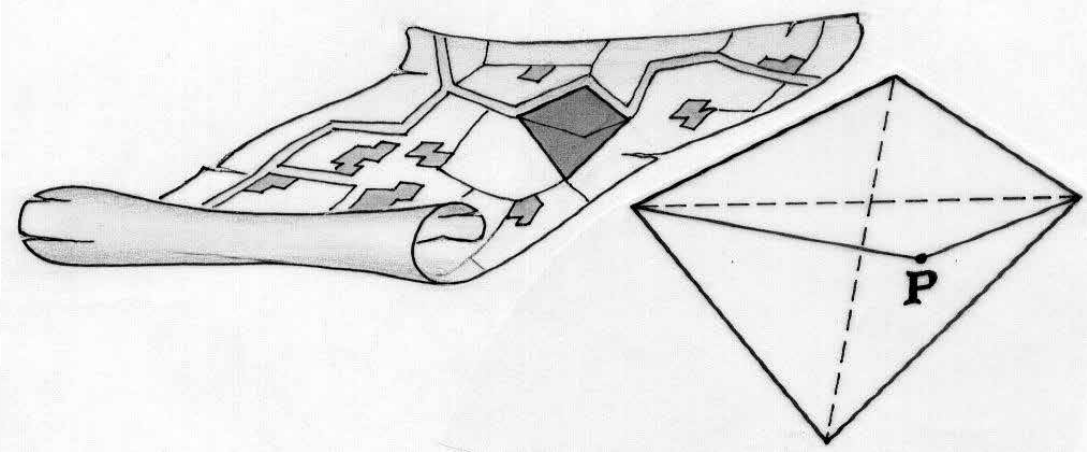
7 MARKS

Old Jacques wants to divide his field into two equal areas when he leaves them to his two sons Pierre and Paul. The field is a quadrilateral.

Pierre says: "There is a straightforward way to do that. Just choose the correct point P on one of the diagonals and join it to the ends of the other diagonals."

Paul adds: "That's true but if you allow P to move off the diagonal you can find an infinite number of possibilities for P."

Draw a quadrilateral to represent Old Jacques' field.
Show clearly the position of P that Pierre has said is the solution. Explain why the two areas are equal.
Draw the complete range of solutions that Paul suggests. Explain your answer.



Question 6 Get back to the beginning

5 MARKS

Give your answer in French, German, Italian or Spanish using at least 30 words.

Alex, Claude et Sam jouent. A la fin de chaque manche, le perdant donne une partie de ses jetons aux deux autres joueurs pour que ceux-ci doublent chacun leur nombre de jetons. La cinquième manche est achevée, Alex possède 10 jetons, Claude en a 9 et Sam seulement 8.

Trouver le nombre de jetons que chaque joueur possédait avant de commencer à jouer. Expliquer.

Alex, Claudius und Sam machen ein Spiel. Am Ende jeder Runde gibt der Verlierer einen Teil seiner Spielsteine an seine beiden Mitspieler, und zwar so, dass sich deren jeweilige Anzahl an Spielsteinen verdoppelt. Nach der fünften Runde besitzt Alex 10, Claudius 9 und Sam nur 8 Steine.

Findet die Anzahl an Spielsteinen heraus, die jeder Spieler vor Spielbeginn hatte.

Question 6 Get back to the beginning

5 MARKS

Alessandro, Claudio e Samuele giocano. Alla fine di ogni mano chi perde dà una parte dei suoi gettoni agli altri due giocatori in modo che questi raddoppino ciascuno il numero dei loro gettoni.
Alla fine della quinta giocata Alessandro possiede 10 gettoni, Claudio ne ha 9 e Samuele solamente 8.

Individuate il numero di gettoni che ogni giocatore aveva prima di cominciare a giocare.
Motivate la risposta.

Jorge, César y Roberto están jugando. Al final de cada mano, el perdedor les da una parte de sus fichas a los otros dos jugadores para que cada uno doble su número de fichas. Se ha terminado la quinta mano, Jorge tiene 10 fichas, César tiene 9 y Roberto sólo 8.

Encuentra el número de fichas que posee cada jugador antes de empezar a jugar.
Razona tu respuesta.

Question 7 True/false

7 MARKS

I am a whole number greater than 2. In this list of paired sentences there is always one true statement about me and one false one in each pair.

1a. I am a number with two digits.
1b. I am an even number.

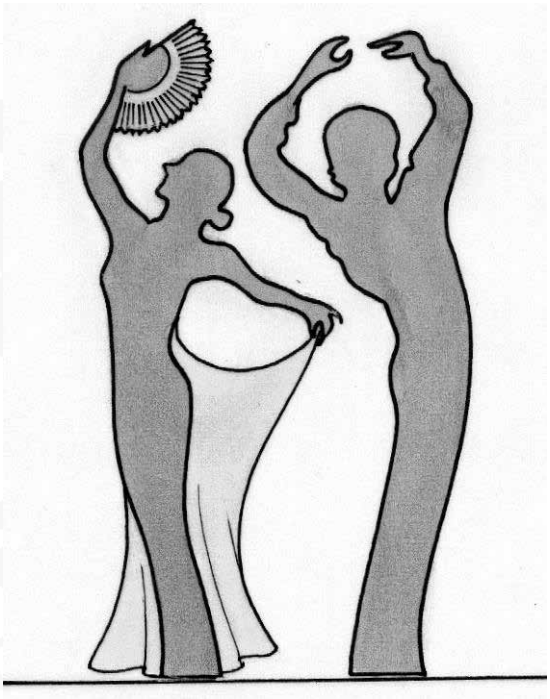
2a. I am the square of a whole number.
2b. I am a number with three digits.

3a. I am a number which contains a 7.
3b. I am a number which has only two divisors, 1 and myself.

4a. I am the product of two consecutive odd numbers.
4b. I am equal to the square of a whole number plus one.

5a. I am divisible by 11.
5b. I am equal to the cube of a whole number plus one.

Who am I? Explain your answer

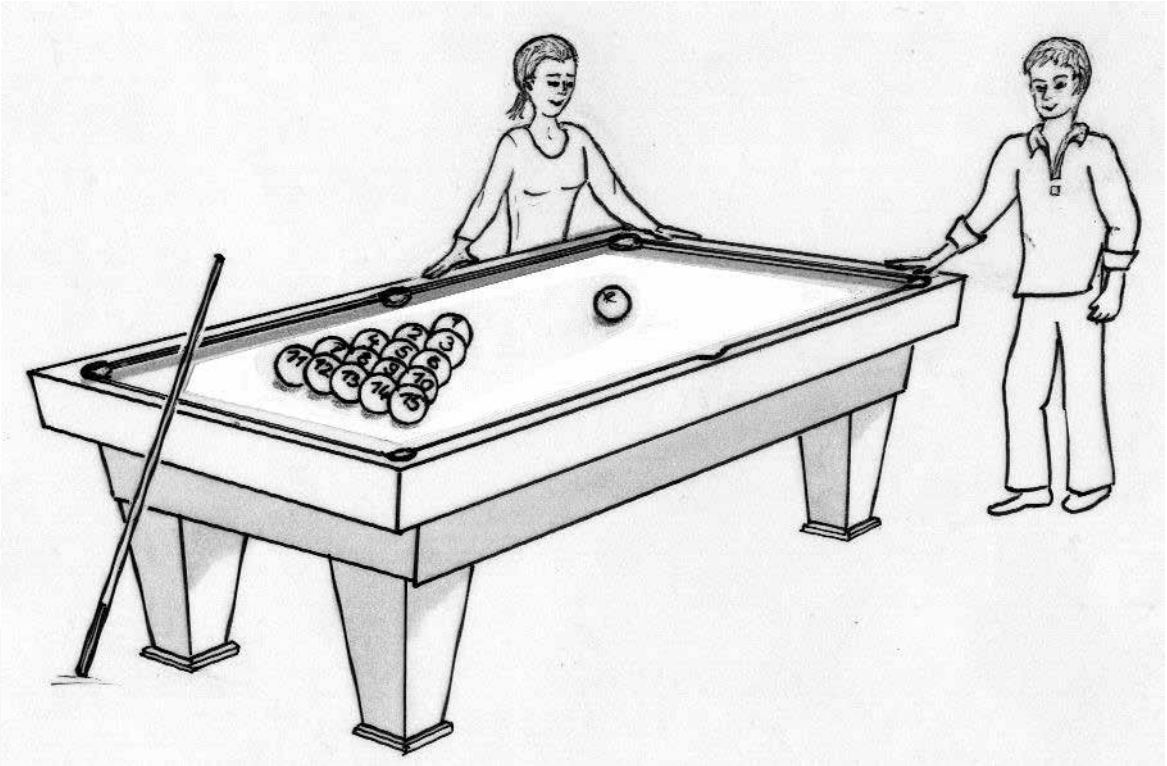


Question 8 Pot black

5 MARKS

American billiards is played with 15 balls numbered with a value of 1 to 15 and one white ball. The game is over when only the white ball is left. At the end of the game Bonnie and Clyde count their points for the balls they have potted. All the balls are counted by one or other of them. Bonnie has exactly twice the points that Clyde has even though she has potted fewer balls than he did.

Find the different ways that Bonnie can score her points.

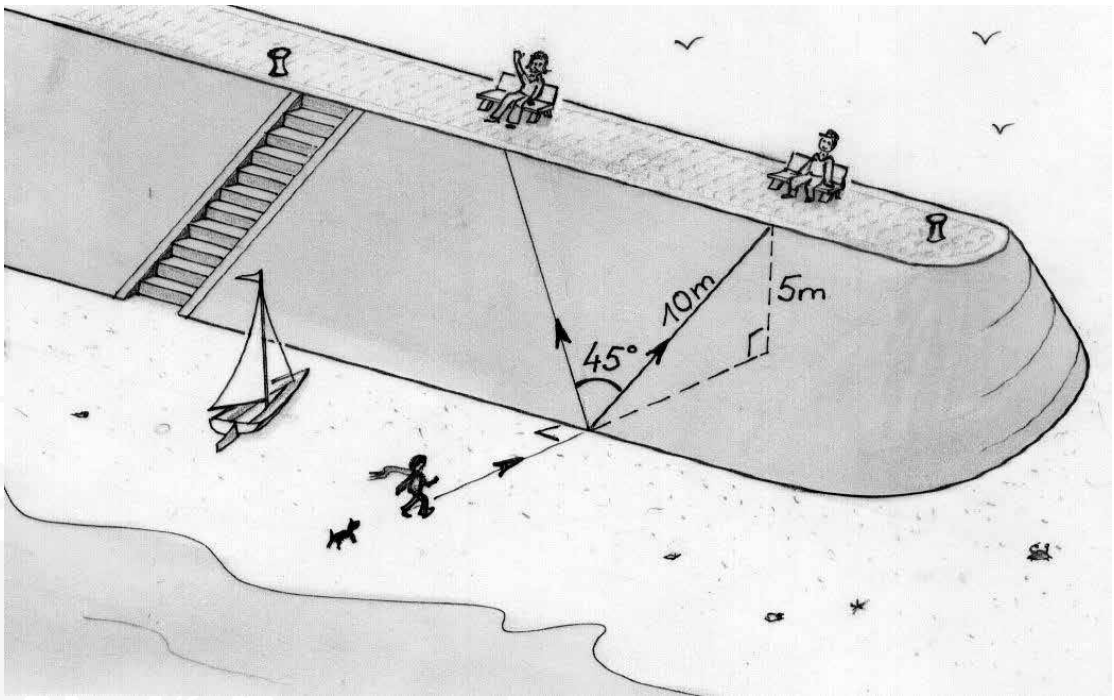


Question 9 See wall

7 MARKS

Lily walks along the beach to climb up the protective sea wall at Malo-les-Bains. The wall is 5 metres high. The shortest path, which is also clearly the steepest, is 10 metres long. The slope for this path is 5 out of 10, 50%.
As she was tired she decided to climb a path which made an angle of 45° with the shortest path.

Find the slope of this path as a percentage.
If Lily wanted to climb a 25% slope, what angle would that path need to make with the shortest one?



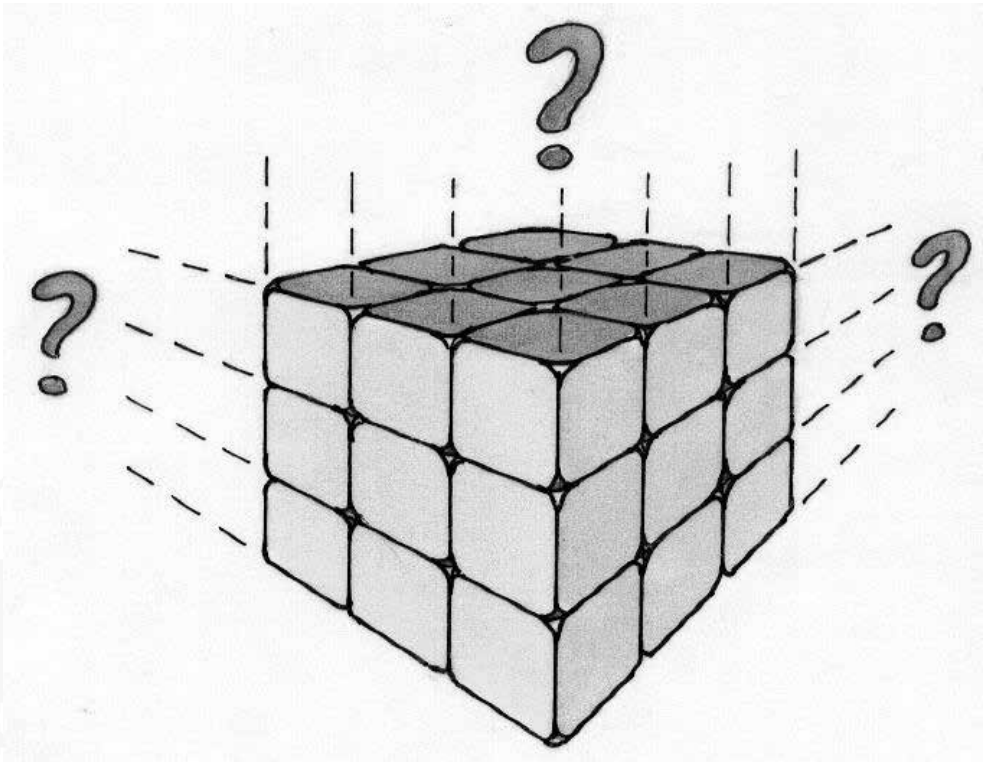
Question 10 Face painting for beginners 10 marks

10 MARKS

A number of small cubes of 1 cm^3 are assembled to make one large cube. Some but not all of the faces of the large cube are then painted completely over. The result is that exactly 48 of the small cubes in the overall assembly are left without a single painted face.

Find all the possible large cubes that meet these conditions. Explain your answer.

In each case draw the net of the larger cube and show clearly which faces are painted.



2013

Question 11 Special committee

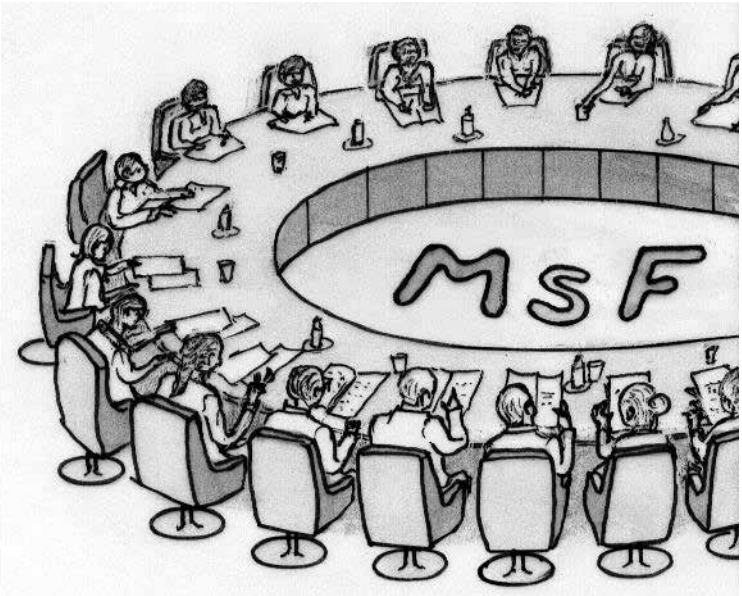
5 MARKS

Senior classes only

During the annual committee meeting of Mathématiques Sans Frontières, the committee members sit at a large circular table. The committee has men and women members.
7 women have a woman to their right and 12 women have a man to their right.
3 men out of 4 have a woman on their right.

From the people at the meeting, one is chosen at random to take the minutes.

What is the probability that a woman will be chosen? Explain your answer.



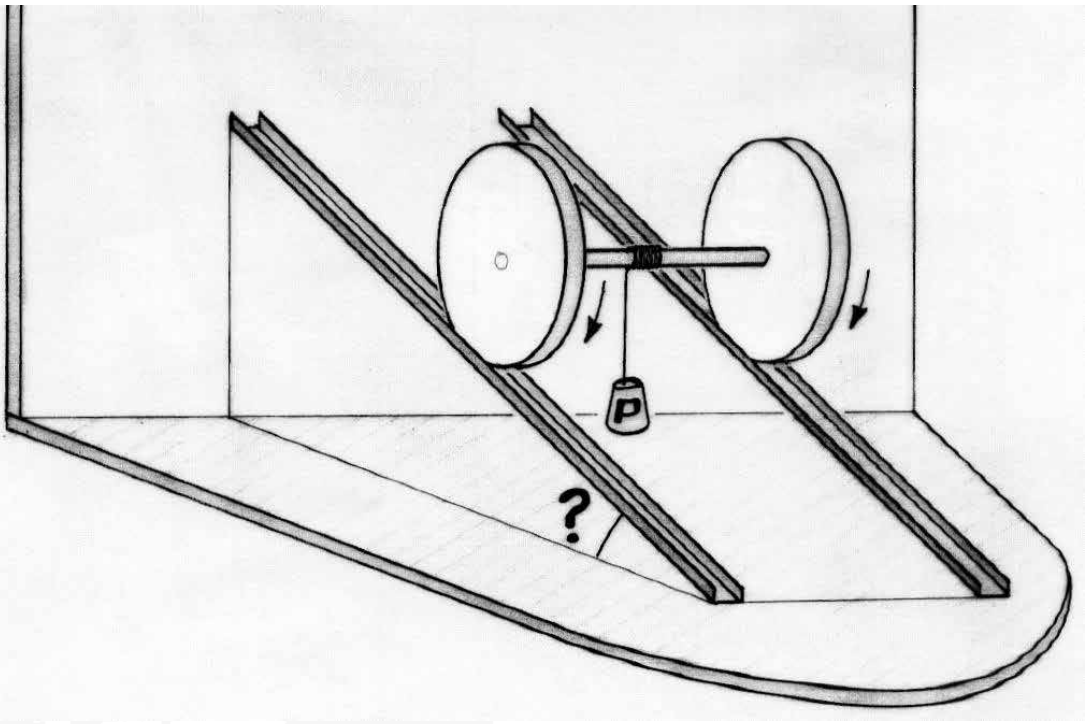
Question 12 Slippery slope

10 MARKS

Senior classes only

The figure shows two wheels linked by an axle. The wheels roll without slipping on two sloping parallel rails. A string which supports a weight is wrapped round the axle. As the wheels move down the incline the string wraps round the axle ensuring that the weight moves horizontally only.
The diameter of the wheels is 10 cm and the diameter of the axle is 1 cm.

Work out the angle of the rails to the horizontal.



2013

Question 13 And so it was inscribed...

10 MARKS

Senior classes only

Anna is hoping to find all the right-angled triangles that meet these two conditions:

- their sides are a whole number of centimetres
- the radius of the circle inscribed inside the triangle is 4 cm.

To succeed in this she uses her knowledge of geometry to construct the figure shown.

Find all the triangles that satisfy the two conditions. Justify your answer.