

# Maths Without Borders – Finland

December 2012

- During the competition, you are not allowed to use cellphones.
- Use only one answer sheet for each question.
- Every solution, even uncomplete, will be taken into account.
- The competition lasts 2 hours.

## Question 1. Tour de chien

7 marks

Solution à rédiger en allemand, anglais, espagnol ou italien en un minimum de 30 mots.

Mein Hund und ich starten gemeinsam in gleicher Richtung zu einer Tour um den See. Wir nehmen den gleichen Weg, jeder mit gleichbleibender Geschwindigkeit. Aber mein Hund läuft schneller als ich und überholt mich einmal bevor wir wieder gleichzeitig an unserem Ausgangspunkt ankommen.

*Wenn nun mein Hund mit der gleichen Geschwindigkeit, aber in entgegengesetzter Richtung den See umrundet hätte, wie viele Male hätte er dann meinen Weg gekreuzt? Begründet eure Antwort.*

My dog and I set out together to go round the lake. We left at the same time from the same starting point, we both took the same path in the same direction and we travelled at a constant speed. But my dog goes much faster than I do and he passed me once before we arrived back at the same time at the starting point.

*What if my dog had gone round the lake at the same speed but in the other direction, how many times would he have passed me? Explain your answer.*

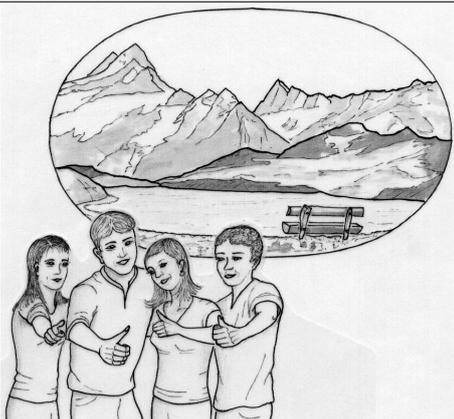
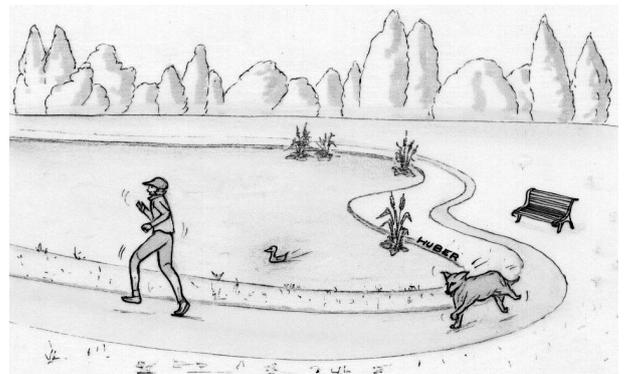
Mi perro y yo salimos juntos y en el mismo sentido para dar la vuelta al lago. Tomamos el mismo camino, cada uno a una velocidad constante. Pero mi perro va bastante más rápido

que yo y me adelanta una vez antes de que lleguemos los dos, en el mismo instante, a nuestro punto de partida.

*Y si mi perro hubiese dado vueltas alrededor del lago a la misma velocidad, pero en sentido contrario, ¿cuántas veces me cruzaría con él? Explicalo.*

Il mio cane ed io ci avviamo assieme per fare il giro del lago nel medesimo senso. Imbocchiamo il medesimo cammino, ciascuno con una velocità costante. Il mio cane, però, è più veloce di me e mi supera una volta prima che si arrivi assieme nello stesso istante al punto della nostra partenza.

*E se il mio cane avesse girato attorno al lago alla stessa velocità precedente, ma in senso inverso, quante volte mi avrebbe incrociato? Spiegate la risposta.*



## Question 2. That's that settled

5 marks

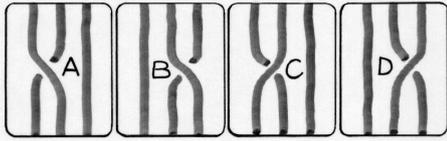
Four friends – Louise, Mylene, Jules and Cissé are back from their holidays. They travelled in a car and now have to settle up the costs involved. Louise paid for the petrol : 96 €.

Mylène paid for the motorway tolls : 42 €.

Jules paid for everyone's snacks on the journey 18 €.

Cissé lent Jules 15 € to help him buy a present for his mother. The four friends want to divide up the costs fairly making the smallest number of transactions.

*Explain how they will do that.*



**Question 3. Plaits – easy as ABC!**

**7 marks**

Cindy and Pierrick have a software package that allows them to create all the different plaits that are possible with three strands of hair.

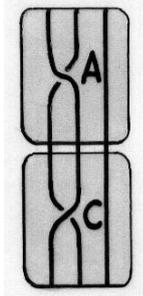
The plaits are created by a sequence of operations from the four operations A, B, C, D shown here.

They notice that C cancels out A because the sequence AC gives three parallel strands if you pull them tight.

*Write down all the sequences of operations which cancel each other out. Pierrick inputs the sequence DDACBAAACDDCABABD by tapping randomly on the key board.*

*Give a sequence of five operations which could be written after Pierrick's sequence which would undo the plait he created.*

*Easy as ABC !*

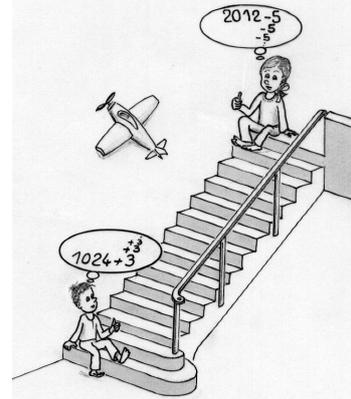


**Question 4. Counting on you**

**5 marks**

Michèle and Michel count at the same time and to the same rhythm. Michèle starts at 2012 and goes down by 5 each time : 2012, 2007, 2002, 1997 ... Michel starts at 1024 and goes up by 3 : 1024, 1027, 1030, 1033 ...

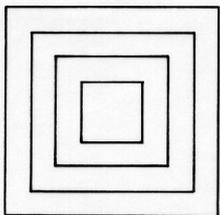
*What are the closest numbers that they will both say at the same time? Explain your answer.*



**Question 5. After the rain**

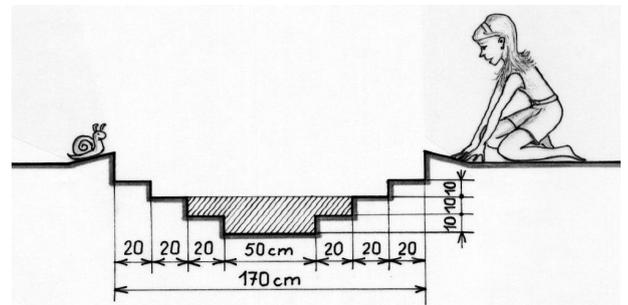
**7**

**marks**



Coralie has built a garden pond. It has little steps as shown. The bottom is a square of side 50 cm. The three steps each have a height of 10 cm and a depth of tread 20 cm as shown. The base is horizontal. Suddenly a storm starts and torrential rain starts to fall. After the rain the good weather comes back and Coralie notices that the water level reaches the second step.

*What volume of water in litres per m<sup>2</sup> fell during the storm? Justify your answer.*



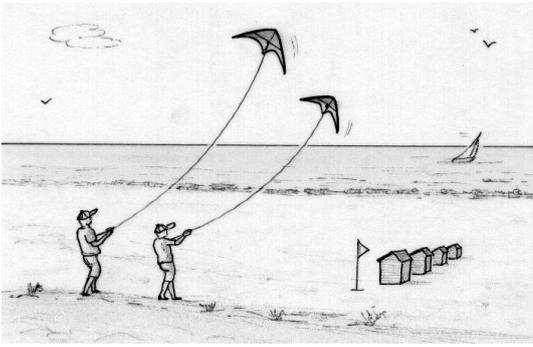
**Question 6. Squaring up**

**5 marks**

The diagram shows a rectangle made up of 13 squares. The lengths of the sides of the squares are a whole number of millimetres. All the squares are different sizes. Each of the four numbers shown is the length of the side of the square in which it has been placed.

*Find the lengths of the sides of the other squares.. Draw a larger version of the diagram and show the length of side for each square.*





**Question 7. Flying a kite**

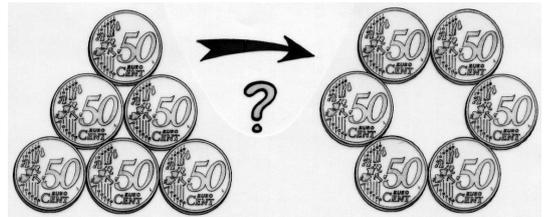
**7 marks**

Draw an equilateral triangle ABC inscribed in a circle of radius 8 cm. P is a point on a side of the triangle ABC. The line through P perpendicular to BC cuts the circle at E and F. Mark the points M which is the mid-point of EP and N the mid-point of FP.

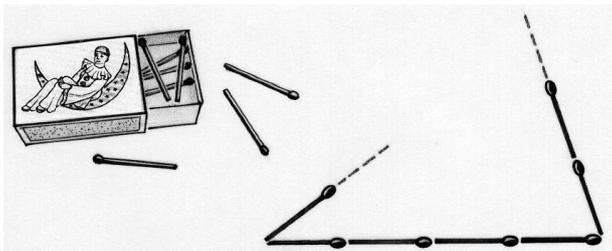
*By marking enough points draw the curves traced out by the points M and N as P moves along the three sides of the triangle ABC.*

**Question 8. Why's Penny so penny-wise?** **5 marks**

Six coins are arranged in a triangle on a table top as shown on the left. Bernard has found a sequence of 5 moves which shifts the coins into the arrangement on the right. A move consists of sliding a coin, without moving any others, to make it touch two others. Penny has found a way of doing it with only 4 moves.



*Draw the six positions for the sequence of 5 moves that Bernard found. Or better, try to find the solution in 4 moves that Penny found and draw the five positions.*



**Question 9. Matching up**

**7 marks**

You have 21 matches that are all the same length. First you make a straight line of some matches putting down one behind another. Then you complete the construction in order to make a triangle using the rest of the matches. Each side of the triangle will be a whole number of matches.

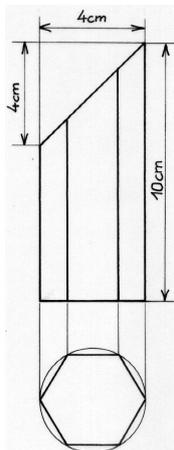
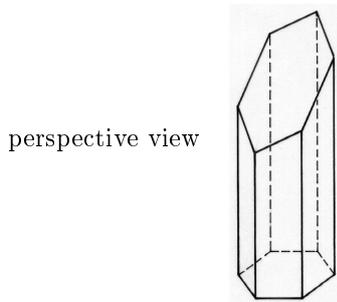
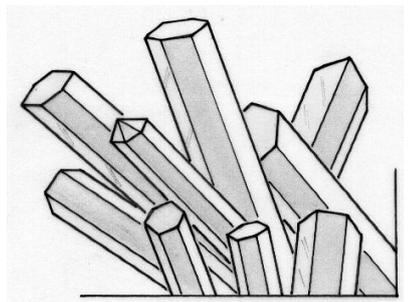
*How many different triangles can you make that use 21 matches? Write down all the possible solutions.*

**Question 10. Crystals**

**10 marks**

Quartz crystals frequently take the shape of hexagonal prisms. The crystal shown below has a regular hexagon of side 2 cm as its base. The prism is cut by a plane at 45° to the base.

*Draw an accurate net of the crystal. It should be full size and show the vertical faces and the two hexagonal faces.*



right view

bottom view



**Question 11. Freedom – on condition**

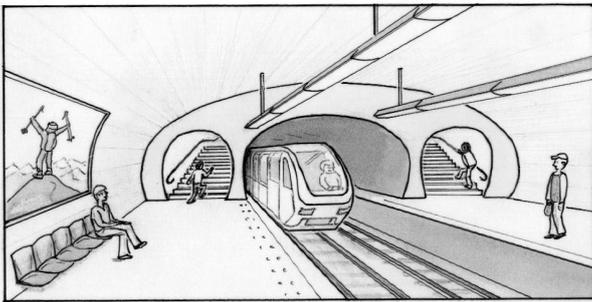
**5 marks**

A prisoner asks to be given one more chance. His jailer gives him some hope of freedom. He brings him two urns, 12 white marbles and 12 black marbles. The prisoner is to share out the 24 marbles between the two urns, putting any number of marbles of any colour he likes in each urn. The jailer will choose an urn at random and then a marble at random from the urn. If the marble is white the prisoner will be freed.

*How should the prisoner share out the marbles in the urns to have the best chance of freedom? Calculate the probability of his freedom with this sharing out.*

**Question 12. Stepping Out**

**7 marks**



While waiting in the metro for his girlfriend Julie, Lucas watches the different ways people climb the exit stairs. He wonders how many different ways you can go up a staircase if at every step you either go up two stairs at a time or just one.

«For a staircase of two or three steps the answer is easily found. But for a stair of 3 steps? Well for the first move there are 2 possibilities, and then I either have 2 steps or three still to go ...»

«Hi Lucas! are you daydreaming» says Julie? He hadn't seen her arrive. «Come on quickly, we're late.»

*How many ways are there to climb a stair of 13 steps taking either one or two steps at a time? Show clearly how you found your answer.*

**Question 13. Foldaway**

**10 marks**

Nicole always thinks ahead: she always carries a folding plastic glass in her handbag - just in case. Her glass is made with a solid base and 5 conical plastic sections. The five sections (figure 1) fold down so that the glass is nearly flat (figure 2). The diagram below shows the glass folded up. The smallest section has interior diameters of 30 and 38 mm. Each section has radii 4mm bigger than the respective radii of the previous, smaller, section. Each section has a height of 20 mm. The thicknesses of the spaces are all the same.

*What is the interior height of the glass when it is unfolded? Explain your answer. You can work out the volume of liquid the glass can hold. Find this volume explaining clearly how you found it. You can use this formula for a section across a cone where the section has radii  $r$  and  $R$  and height  $h$ .*

$$V = \frac{\pi h}{3} (r^2 + rR + R^2) .$$



figure 1



figure 2

