

Matematica Senza Frontiere Junior

13 – 14 year old students

Competition March 12, 2013

Answer Sheet

Exercise n. 1 (7 points) Lie Detector.

The solution must be given in one of the languages proposed with at least 15 words.

There are two possibilities for each box:

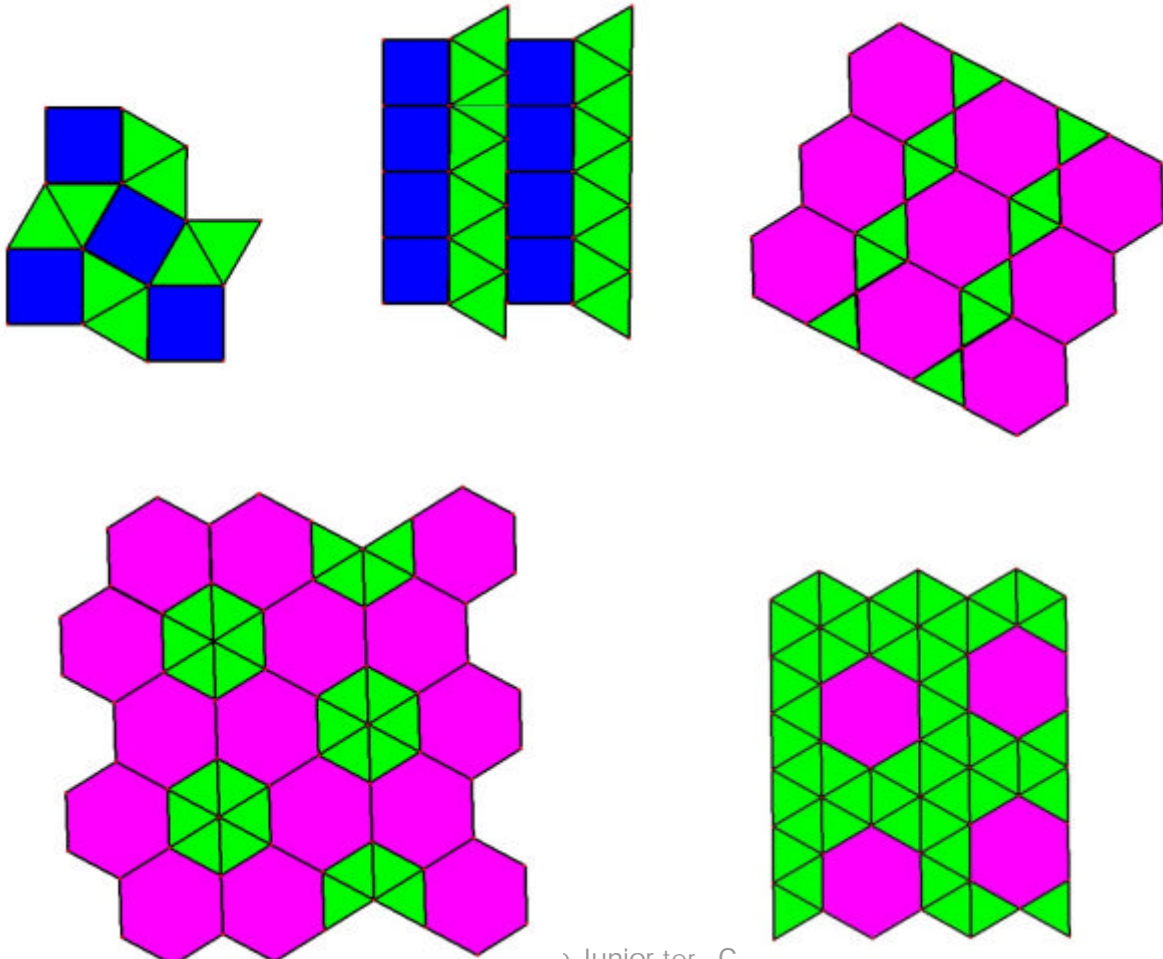
Selecting an animal, for example, from the second box, will determine what the real situation is.

BOX	2 rabbits	1 dove 1 rabbit	2 doves
Possibilities	2 doves	2 rabbits	1 dove 1 rabbit
	1 dove 1 rabbit	2 doves	2 rabbits

Exercise n. 2 (10 points) Happy Floor

If Paola uses two shapes from those indicated, she would probably use the triangles and squares OR the hexagons and triangles.

Some of the possible types of flooring (pavement) patterns are the following :



Exercise n. 3 (5 points) Panoramix and the Magic Potion

Panoramix will be able to serve his super-soup at 11:00 pm on October 31 .
In fact, 314 hours : 24 hours = 13 days + 2 hours

Exercise n. 4 (7 points) Triangles everywhere

The perimeter will be at most 111 cm given that the drumstick must be an integer number which is smaller than the sum of the other two drumsticks.

Exercise n. 5 (10 points) The Fibonacci Tree

The number of branches at the 20th level is **6765**.
In fact, the Fibonacci sequence is given by the following numbers, (until the 20th level):
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, **6765**,....

Exercise n. 6 (7 points) Refreshment Stands

Carlo occupies the 4th position and Luigi is 1 800 meters from the finish line.

Exercise n. 7 (10 points) Sacred and Profane

The first day of the year coincides every 73 years with the sacred calendar and every 52 years with the agricultural calendar.
LCM (260 and 365) = 18 980 18 980 : 260 = 73 18 980 : 365 = 52

Exercise n. 8 (5 points) Factory Logo

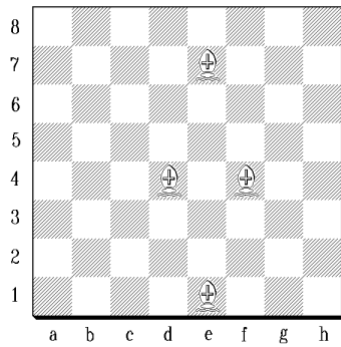
Given r, the radius of the circle, the black square has an area of $4 r^2$, **the grey square $2 r^2$** (the diagonal is equal to the diameter)
Black shading: $4 r^2 - r^2 = 3 r^2$ Ratio = **3/2**

Exercise n. 9 (10 points) Dinner at AnnaMaria's

We immediately observe that the reasoning is symmetric (men and women) .
Let's imagine that the first person in line is a man, work out the possibilities and multiply everything by 2 (just in case the first person in line is a woman).
The friends must sit down, alternating male-female, hence M-F-M-F-M-F OR sym metrically, F-M-F-M-F-M
For the first position there are 3 possibilities (3 men)
For the second position there are also 3 possibilities (3 women)
For the third position there are now 2 possibilities (the two men remaining)
For the fourth position there are 2 choices (the two women remaining)
For the fifth position there is only one choice (the man remaining)
For the sixth position there is one choice (the last woman).
In total, we have, multiplying all possibilities and then multiplying again by 2
 $2 \cdot (3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1) = 72$

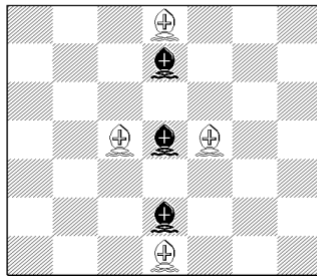
Exercise n. 10 (7 points) Covering with the Bishop

- a) Given that the bishop may move on the white squares or black squares, we can calculate both cases separately by looking at the diagonals. Let's start with the black diagonals and put the bishop on the longest diagonal: only 4 bishops are necessary to dominate all the black squares. For example :



due to symmetry, another 4 bishops are necessary for the white squares. Therefore, 8 bishops are the minimum number necessary to dominate the 8x8 chessboard.

- b) There are not the same number of black and white diagonals.



There are 3x3 white squares and 4x4 black squares, therefore $4+3=7$ bishops are necessary to dominate all the squares.